Indian Statistical Institute, Bangalore

B. Math. Second Year

First Semester - Optimization

Duration : 3 hours Date : January 05, 2018

Back Paper Exam **Total marks: 40**

Each question carries 5 marks

1. Find the QR decomposition of $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$.
2. Prove the singular value decomposition for a matrix.
3. Let A be a matrix with rank r and $A = VDU^*$ be the SVD. Prove that the range of A is spanned by $V_1, \cdots V_r$.
4. Find Perron pair for $\begin{pmatrix} 7 & 2 & 3 \\ 1 & 8 & 3 \\ 1 & 2 & 9 \end{pmatrix}$.
5. Find least square solution to the system $x_1 - x_2 = 2$; $x_1 + x_2 = 4$; $2x_1 + x_2 = 8$.
6. State and prove a sufficient condition for a LP to be unbounded.
7. State and prove minmax Theorem.
8. Solve

$$\begin{array}{ll} \max & 3x_1 + 2x_2 + x_3 \\ \text{subj} & 2x_1 + x_2 + x_3 \leq 150 \\ & 2x_1 + 2x_2 + 8x_3 \leq 200 \\ & 2x_1 + 3x_2 + x_3 \leq 320 \\ & x \geq 0. \end{array}$$